Testing the Altitude and Latitude Based Solar Irradiance Models in Kenya

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ABSTRACT

The OPEC oil embargo of the 1970s sparked off the era of innovative development of renewable energy technologies with solar energy being one of the principal components. A major input in solar energy technology is solar radiation composed of direct and diffuse components whose sum is known as global radiation. In some countries only a few meteorological stations measure solar radiation and this has necessitated more research and development (R&D) of various models that estimate direct, global, and diffuse radiation using measured solar radiation and other meteorological parameters. This paper assesses both Tiwari's Latitude and Gopinathan's Altitude Models derived from the regression of measured clearness index (K_0 *) against fraction of bright sunshine duration (* S/S_0 *) in the classical Angstrom-Prescott type regression model. Measured solar radiation and sunshine duration data from four Kenyan meteorological stations of Dagoretti, Eldoret, JK Airport, and Voi was processed for consistency and statistical quality. The clearness index* $K_0 = \overline{H}/\overline{H}_0$ *and the fraction of bright sunshine* $(\overline{S}/\overline{S}_0)$ *were calculated for each station and curve-fitted to obtain the quadratic form of Angstrom-Prescott equation (* $K_0 = c_1 + c_2(\overline{S}/\overline{S}_0) + c_3(\overline{S}/\overline{S}_0)^2$ *) where* c_1, c_2 *are the coefficients that are altitude dependent on one hand and latitude dependent on the other. Model performance was measured using goodness of fit statistics that included; Pearson correlation coefficient (r), coefficient of determination (* R^2 *), Mean Bias Error (MBE), Root Mean Square Error (RMSE), Students- t-statistic, and the t-test. For each of the four stations investigated the model equation of the form* $K_{alt} = c_1 + c_2(\overline{S}/\overline{S}_0) + c_3(\overline{S}/\overline{S}_0)^2$, $K_{lat} = b_1 + b_2(\overline{S}/\overline{S}_0) + b_3(\overline{S}/\overline{S}_0)^2$ *are presented and discussed. It emerged that out of the four sites assessed, Voi produced the best performing Altitude model (t-statistic = .070) while Eldoret produced the best overall performing Latitude model (t-statistic = .547). It was recommended that the 8 models produced could be used to estimate the irradiance at these sites with the fraction of sunshine duration being considered to be the only input.*

Keywords**:** Solar Radiation, Goodness of Fit Statistics, Angstrom-Prescott, Sunshine duration.

INTRODUCTION

In designing, implementing, or modeling solar energy systems such as photovoltaics, solar water heaters or dryers, a key input parameter is the amount of solar radiation falling on the collector surface and its temporal and spatial variation. For high efficiency, the collector surface may be designed to track the sun but in the majority of cases, the collector is fixed at some optimal orientation. The minimum information required for such a system is the global radiation falling on a horizontal collector surface. Since it is not always possible to make *in-situ* measurements of solar radiation, it has become a tradition to estimate this parameter based on related variables; such as fraction of sunshine duration, cloud cover, ambient temperature, and certain geographical coordinates. In this paper, we report on the testing of two solar radiation estimating models based on three parameters namely, clearness index, latitude of the site, and the corresponding altitude. The two models have been named the *Tiwari Latitude Model* and the *Gopinathan Altitude Model* after the two principal authors who first suggested such models. The main aim of this study was to develop models that estimate the long-term mean monthly global irradiance over Kenya using the altitude-based model and latitude-based model. To achieve these, the following objectives were set out to:(1) establish which of Kenya's 11 meteorological stations has long-term measured data that has a strong correlation between global solar radiation and sunshine duration, (2) determine the clearness index $K_0 = \overline{H}/\overline{H}_0$ and fractional sunshine duration ($\overline{S}/\overline{S}_0$) using data from selected stations, (3). fit clearness index and fraction of sunshine duration into the latitude dependent model and the altitude dependent model, and (4) test the reliability and usability of the resulting models.

MATERIALS AND METHODS

Study Location

The geographical details of the four stations used in this study are shown in Table 1 below.

Table 1: Geographical details of the study locations

Station	Latitude (^{0})	Longitude (^{0}E)	Altitude (m)
Dagoretti	-1.3000	36.7500	1798
Eldoret	0.5333	35.2833	2120
JKIA Airport	-1.3167	36.9167	1624
Voi	-3.4010	38.3400	579

Irradiance data from 11 meteorological stations consisting of daily sums and monthly means of global radiation and monthly means of daily sums of bright sunshine duration were downloaded from archives of the National Renewable Energy Laboratory (NREL) Golden, Colorado(USA). Detailed instruction on the use of this data is found in Krichak (1987), a World Meteorological Organization (WMO) document. The data was checked for consistency and completeness of the record before the clearness index and fraction of sunshine duration were determined and correlated. The stations for which clearness index and fraction of sunshine duration correlated strongly as measured by the coefficient of determination (R^2> .640) or Pearson correlation coefficient ($r \phi .800$) were selected for the study and these were Dagoretti, Eldoret, and Jomo Kenyatta Airport. For each of these stations, the clearness index K_0 was regressed against corresponding fraction of bright sunshine duration $\bar{S}/\bar{S}_0$ to give a quadratic relationship $K_0 = c_1 + c_2\left(\bar{S}/\bar{S}_0\right) + c_3\left(\bar{S}/\bar{S}_0\right)^2$ where the regression coefficients are latitude dependent on one hand and altitude dependent on the other. These quadratic models were tested for long-term performance by the use of the Mean Bias Error (MBE), short-term performance using the Root Mean Square Error (RMSE) and overall performance using the t-statistic (t-stat). A good description of these model performance metrics can be found in Zhang *et al*, (2014).

Theory

The clearness index K_0 is ratio of the monthly mean of daily sum of solar radiation $\bar{H}$ measured on a horizontal surface located at the collector site, to the monthly mean of daily sum solar radiation $\bar{H}_0$ falling on the extraterrestrial (hypothetical top of the atmosphere) horizontal plane; located directly above the collector. Extraterrestrial daily sum $\bar{H}_0$ is calculated as the integral of the instantaneous irradiance G_0 from sunrise to sunset (sunrise hour angle ω_{sr} = sunset hour angle ω_{ss})

$$H_0 = \int_{\omega_{ss}}^{\omega_{ss}} G_0 dt = \frac{24x3600}{\pi} G_{sc}\left\{1 + 0.033\cos\left(\frac{360n}{365}\right)\right\} \times \left(\cos\phi\cos\delta\sin\omega_{ss} + \frac{\pi\omega_{ss}}{180}\sin\phi\sin\delta\right) \quad (1)$$

The mean irradiance G_{sc} on the extraterrestrial surface facing the sun has been measured by artificial satellites and is generally accepted as 1397 Wm^{-2}. For a horizontal surface this irradiance is modified by Earth's rotation (sunrise to sunset) represented by the hour angle ω and the seasonality which is a function of its elliptical orbit and declination δ arising from the axial tilt. These are all incorporated in Equation 1, where n is the ordinal day number (n =1 on January 1 etc), ϕ is the location latitude, 360 is the number of degrees subtended by a circle while 365 is the number of days in a year. The instantaneous extraterrestrial irradiance is given by

$$G_0 = G_{SC}\left\{1 + 0.033\cos\left(\frac{360n}{365}\right)\right\}(\sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega) \quad (2)$$

The seasonal change is a function of solar declination (δ) and ordinal day number (n):

$$\delta = 23.45\sin\left[360\left(\frac{284+n}{365.25}\right)\right] \quad (3)$$

The Earth's polar (rotational) axis is tilted at 23.45^0 to the ecliptic (plane containing Sun and Earth), and it takes 365.25 ephemeris days for the Earth to complete 1 orbit around the Sun.

The monthly mean of daily sunshine duration $\bar{S}/\bar{S}_0$ is the ratio of measured monthly mean of daily hours of bright sunshine (measured by a Campbell-Stokes Sunshine Recorder, a glass ball instrument common in weather stations or photocensors) to the monthly mean of astronomical daylength which is a parameter calculated from Earth-Sun geometry. The astronomical day length for the average day of the month is given as $S_0 = \frac{2}{15}\omega_s$ where the hour angle for sunset or sunrise is given by:

$$\omega_s = \cos^{-1}\left(-\tan\phi\tan\delta\right). \tag{4}$$

The Tiwari Latitude Model

Using only the latitude (ϕ) and fraction of bright sunshine ($\bar{S}/\bar{S}_0$) sunshine Tiwari and others (Tiwari & Saluja, 1997 and Frere *et al*, 1980) modified the quadratic Angstrom-Prescott equation ($\bar{H}/\bar{H}_0 = a + b(\bar{S}/\bar{S}_0)$) by expressing its coefficients as:

$$a = -0.110 + 0.235\cos\phi + 0.323(\bar{S}/\bar{S}_0) \tag{5}$$

$$b = 1.449 - 0.553\cos\phi - 0.694(\bar{S}/\bar{S}_0) \tag{6}$$

This leads to the quadratic form:

$$K_{lat} = b_1 + b_2\left(\bar{S}/\bar{S}_0\right) + b_3\left(\bar{S}/\bar{S}_0\right)^2 \tag{7}$$

The Gopinathan Altitude Model

Gopinathan (1988) used $\bar{S}/\bar{S}_0$ and the altitude (Z) in km to derive Eqn. (10) and in this latter case, the Angstrom-Prescott coefficients are as given by:

$$a = 0.265 + 0.070Z - 0.135(\bar{S}/\bar{S}_0) \tag{8}$$

$$b = 0.401 - 0.108Z + 0.325(\bar{S}/\bar{S}_0) \tag{9}$$

This results in the quadratic equation:

$$K_{alt} = c_1 + c_2\left(\bar{S}/\bar{S}_0\right) + c_3\left(\bar{S}/\bar{S}_0\right)^2 \tag{10}$$

Recent reviews of the Latitude and Altitude models are found in the literature (see for example Namrata, Sharma, and Saksena, 2013; Ahmad and Tiwari, 2010; Katiyar and Pandey, 2013).

Procedures

Using MATLAB curve-fitting routines data from the four stations Dagoretti (DAG), Eldoret (ELD), JK Airport (JKA), and Voi (VOI) were processed to yield the quadratic curves of Equations (7) and (10) above. These quadratic models were then subjected to goodness of fit statistics whose outcomes are shown in Table 3 and Table 4.

RESULTS AND DISCUSSION

The curve fitting coefficients for Eqn. (7) and Eqn. (10) are shown in Table 2 below.

Table 2 Curve Fitting Coefficients for Latitude and Altitude Models

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Coefficient	Dagoretti	Eldoret	JK Airport	Voi
b_1	0.110	0.092	0.051	0.340
b_2	0.960	1.300	1.600	2.300
b_3	0.360	0.690	0.690	0.690
c_1	0.390	0.410	0.380	0.310
c_2	0.072	0.037	0.091	0.250
c_3	0.320	0.320	0.330	0.330

Results of the goodness of fit statistics tests are summarized in Table 3 for the Latitude Model and Table 4 for the Altitude Model respectively.

Table 3 Goodness of Fit Statistics for Latitude (Tiwari) Model.

Loc	r	R^2	RMSE	MBE	t-stat	t-test
DAG	0.821	0.674	0.571	0.060	1.494	41.539
ELD	0.881	0.776	0.642	0.023	0.547	60.073
JKA	0.900	0.810	0.557	0.138	2.107	45.397
VOI	0.871	0.758	0.609	0.254	5.375	42.053

Table 4 Goodness of Fit Statistics for Altitude (Gopinathan) Model

Loc	r	R^2	RMSE	MBE	t-stat	t-test
DAG	0.850	0.723	0.571	0.025	0.607	43.067
ELD	0.874	0.764	0.642	-0.033	0.782	56.854
JKA	0.891	0.794	0.557	0.066	1.148	41.323
VOI	0.864	0.746	0.609	0.009	0.170	39.768

The independent variable in respect of the statistics in both the Latitude and Altitude model is the measured duration of sunshine fraction ($\bar{S}/\bar{S}_0$) while the dependent variable is the estimated clearness index ($\bar{H}/\bar{H}_{lat}$) and ($\bar{H}/\bar{H}_{alt}$) respectively.

The Latitude Model

A low RMSE is desired for short-term performance and by this criterion the best performing model was for JK Airport (*RMSE = .557*), followed by Dagoretti (*RMSE = .571*), Voi (*RMSE = .609*), and Eldoret (*RMSE = .642*).

For long-term performance a low MBE value is desired. The lowest positive MBE was observed for Eldoret (*MBE = .023*), followed by Dagoretti (*MBE = .060*), J K Airport (*MBE = .138*), and Voi (*MBE = .254*). Eldoret, with the lowest MBE produced the best long-term performing model. The criterion for the model with the best overall performance is the t-statistic. In other words, given observed fraction of bright sunshine duration ($\bar{S}/\bar{S}_0$) as the input, the t-statistic tell us which model returns values of latitude-based clearness index (K_{lat}) that best simulates measured clearness index (K_0). The lowest value obtained was for Eldoret *(t-stat = .547*), followed by Dagoretti (*t-stat = 1.494*), J K Airport (*t-stat = 2.107*), and Voi (*t-stat = 5.375*). The overall best performing latitude model was therefore that obtained from Eldoret station.

The Altitude Model

It was observed that this model had exactly the same RMSE characteristics as the Latitude model as observed by comparing Tables 4 and 5. All other statistical parameters were different. The best short-term performing model was that for J K Airport (*RMSE = .557*), followed by Dagoretti (*RMSE = .571*), Voi (*RMSE = .609*), and Eldoret (*RMSE = .642*). This was the case with the Altitude Model as already noted.

The best long-term performer was, according to the calculated mean bias error, Voi (*MBE = .009*), followed by Dagoretti (*MBE = .025*), and J K Airport (*MBE = .066*). As the mean bias error values were positive all these overestimated. Eldoret (*MBE = -.033*) had a negative value and therefore underestimated. The lowest t-statistic value for this model was that for Voi (*t-stat = .170*), followed by Dagoretti (*t-stat = .607*), then Eldoret (*t-stat = .782*), and lastly, J K Airport (*t-stat = 1.148*). The best overall performing Altitude Model was therefore the one obtained with data from Voi station.

Latitude and Altitude Model Equations

The resulting model equations are shown below. These can be used to estimate the clearness index at the site and its neighbourhood using fraction of sunshine duration which is implicitly related to the location altitude and latitude.

Dagoretti:

$$K_{lat} = 0.11 + 0.96\left(\frac{S}{S_0}\right) - 0.36\left(\frac{S}{S_0}\right)^2, \text{ and } K_{alt} = 0.39 + 0.072\left(\frac{S}{S_0}\right) + 0.32\left(\frac{S}{S_0}\right)^2 \tag{11}$$

Eldoret:

$$K_{lat} = 0.092 + 1.3\left(\frac{\bar{S}}{\bar{S}_0}\right) - 0.69\left(\frac{\bar{S}}{\bar{S}_0}\right)^2, \text{ and } K_{alt} = 0.41 + 0.037\left(\frac{\bar{S}}{\bar{S}_0}\right) + 0.32\left(\frac{\bar{S}}{\bar{S}_0}\right)^2 \tag{12}$$

J K Airport:

$$K_{lat} = -0.051 + 1.6\left(\frac{\bar{S}}{\bar{S}_0}\right) - 0.69\left(\frac{\bar{S}}{\bar{S}_0}\right)^2, \text{ and } K_{alt} = 0.38 + 0.091\left(\frac{\bar{S}}{\bar{S}_0}\right) + 0.33\left(\frac{\bar{S}}{\bar{S}_0}\right)^2 \quad (13)$$

Voi:

$$K_{lat} = -0.34 + 2.3\left(\frac{\bar{S}}{\bar{S}_0}\right) - 0.69\left(\frac{\bar{S}}{\bar{S}_0}\right)^2, \text{ and } K_{alt} = 0.31 + 0.25\left(\frac{\bar{S}}{\bar{S}_0}\right) + 0.33\left(\frac{\bar{S}}{\bar{S}_0}\right)^2 \quad (14)$$

CONCLUSIONS AND RECOMMENDATIONS

The overall aim of the study was to develop models for predicting the monthly mean values of daily irradiance at the four meteorological stations in Kenya using latitude and altitude linked monthly means of daily fraction of sunshine duration as the independent variable. All four stations' data was found suitable to be used in the Latitude Model for estimating K_{lat}. Data from JK Airport produced the model with the best short-term performance and Eldoret data produced the best long-term performing as well as the overall best performing model. Data from the same stations was also suitable for use in the Altitude Model for estimating K_{alt}. The best short-term model was obtained with data from JK Airport, the best long-term model with data from Voi, and the best overall performing model obtained with Voi data.

All models derived from data from Dagoretti, Eldoret, JK Airport, and Voi and expressed in the paired equation form (Eqn. 11 to Eqn. 14) may be used to estimate the clearness indices, K_{lat}, and K_{alt} using fraction of sunshine duration $\bar{S}/\bar{S}_0$ as the only input. In the specific circumstances where long-term performance is desired, such models should be used where applicable. The same applies to short-term performing models. The overall best performing models should be used when long-term and short-term performances are not important factors.

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Bio

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